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A METER OF THE TIME OF CAPILLARY SOAKING

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A description of a microprocessing instrument that measures the time of capillary soaking and the cosine of the wetting angle at a horizontal position of a porous body is presented. Determination of the wetting angle and kinetics of capillary soaking is based on approximation of the solution of the differential equation of liquid flow in a capillary with account for evaporation.

At present, processes related to batching of reactants that violate the stability of disperse systems have received wide acceptance in industry. One such process is batching of coagulants in cleaning of sewage and natural waters, optimization of which is very urgent due to the high cost of reactants. Methods of optimum batching of coagulants by the minimum of the time of capillary soaking by using a sheet of testing filter paper are known [1, 2]. In this case, time is the only measured parameter, the value of which is greatly affected by evaporation from the surface, the mean radius of the capillary, and the geometric dimensions of the tested porous body, which makes the results of different measurements incomparable and impedes determination of the minimum value of the time of capillary soaking correlating with an optimum dose of reactants.

The device developed for measuring the time of capillary soaking, the block diagram of which is given in Fig. 1, eliminates the drawbacks mentioned and broadens the range of measurement data, thus allowing one to determine the cosine of the wetting angle by the adopted mathematical model of a porous body.

The device functions as follows. Into the cell with liquid to be analyzed 1 porous material 5 is submerged horizontally; constant sensors of propagation of the wetting boundary 2, 3, and 4 are positioned along the material. An assembly of comparators 6 registers the sequential arrival of the front of liquid propagation l at the position of the electrodes. Noise and false operations are eliminated by the unit controlling the order of operations 7. The unit determining time intervals 8 measures the time intervals between operation of the position sensors 2, 3, and 4 and transmits these data to the unit determining the wetting angle 9 and the unit determining the minimum 10. Results of the calculations are output to the display unit 11.

Water separation of the medium is studied by the time of wetting of two adjacent sections, which allows determination of three times in one experiment. The position of the initial point of the first section is selected for reasons of minimum influence on measurement of dynamic changes of the wetting angle. The lengths of the sections are selected such that in any water separation the boundary of wetting always reaches the end of the second section (see Fig. 1). Due to the anisotropy of the parameters of the testing paper, measurements must be made in several directions relative to the sheet. It is found experimentally that the anisotropy is maximum in two orthogonal directions (along and across the longitudinal axis of the paper roll). Two identical measuring rules are used for estimation of the anisotropic properties of the testing paper.

The main requirement imposed upon the model of the wetted porous body is the identity of calculated and experimental velocities of the boundary of wetting. Since at present there is no generally accepted universal model of a capillary-porous body, we used the simplest of the known notions of filter paper as a set of straight cylindric and parallel capillaries of constant radius and equal wall thickness. In this case, anisotropy of the paper, intersection of capillaries, radius distribution function, and tortuosity of the capillaries and roughness of their walls are disregarded, which makes it possible to ignore the corresponding phenomenological coefficients. In order to allow for evaporation,

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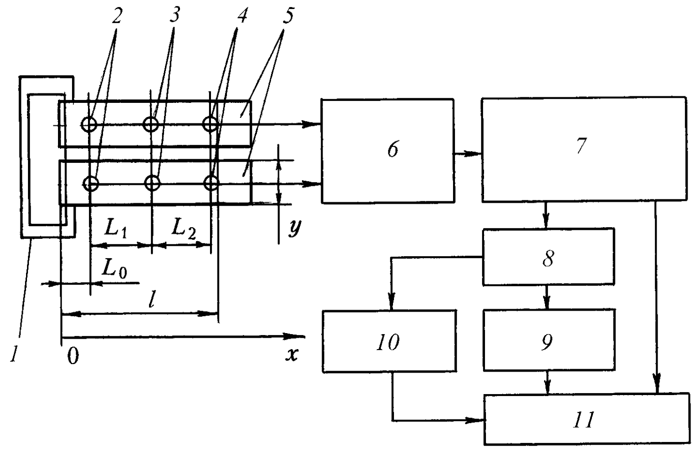

Fig. 1. Block diagram of the meter of the velocity of soaking: 1) cell with analyzed liquid; 2) sensors at the beginning of the first section; 3) sensors at the end of the first section and beginning of the second section; 4) sensors at the end of the second section; 5) testing paper; 6) assembly of comparators; 7) unit controlling the order of operations, the elimination of rattling, and false operations; 8) unit determining time intervals; 9) unit determining the wetting angle; 10) unit determining the minimum; 11) display unit.

we can take the capillary surface to be permeable. The derivation of the mathematical model of the process of wetting is based on the law of momentum conservation for bodies of a variable mass:

$$\frac{dmV}{dt} = F + V\frac{dm}{dt}\,. \tag{1}$$

We determine the total momentum of liquid in the volume of the wetted section at time instant t. The momentum of the liquid layer of length Δx that is at a distance x from the point of contact between the body and liquid is expressed as

$$m_{\Delta x}\, V(x, t) = \rho \Delta x y h \Pi V(x, t)\,. \tag{2}$$

In time Δt, liquid with mass $\Delta m_{\rm en}(x, t) = \rho y h \Pi V(x, t)\Delta t$ enters the section of length Δx that lies at a distance x from the contact point and liquid with mass $\Delta m_{\rm out}(x + \Delta x, t) = \rho y h \Pi V(x + \Delta x, t)\Delta t$ runs out. With $K_{\rm ev}$ being constant along the length of the wetted section and evaporation from the side edges of the porous body being neglected, the mass evaporated from the surface of this section in time Δt is presented in the form $\Delta m_{\rm ev} = 2\rho \Delta x y \Delta t K_{\rm ev}$. Then, from $\Delta m_{\rm ev} = \Delta m_{\rm en}(x, t) - \Delta m_{\rm out}(x + \Delta x, t)$ we obtain $\dfrac{\partial V(x, t)}{\partial x} = -\dfrac{2K_{\rm ev}}{h\Pi}$, whence

$$V(x, t) = -\frac{2K_{\rm ev}}{h\Pi}x + \frac{2K_{\rm ev}}{h\Pi}l + V(l, t) = \frac{2K_{\rm ev}}{h\Pi}(l - x) + \frac{dl}{dt}\,. \tag{3}$$

With account for (2) and (3), the total momentum of liquid along the whole length l of the wetted section

$$P(t) = \int_0^l \rho y h \Pi \left(\frac{2K_{\rm ev}}{h\Pi}(l - x) + \frac{dl}{dt}\right)dx = \rho y h \Pi \left(\frac{2K_{\rm ev}}{h\Pi}\frac{l^2}{2} + l\frac{dl}{dt}\right)$$

allows one to transform the left-hand side of Eq. (1) to the form

$$\frac{d\,(mV)}{dt} = \frac{dP\,(t)}{dt} = \rho y h \Pi \left(\frac{2K_{ev}}{h\Pi}\, l \frac{dl}{dt} + \left(\frac{dl}{dt}\right)^2 + l \frac{d^2 l}{dt^2} \right). \tag{4}$$

In time Δt, evaporation from the section of length Δx, which lies at a distance x from the wetting point, at a liquid velocity on the section $V(x, t)$ gives momentum variation $V(x, t)\Delta m_{ev} = V(x, t)2\rho\Delta xy\Delta tK_{ev}K_s$, whence $V(x, t)\frac{\partial m_{ev}}{dt} = V(x, t)2\rho y\Delta xK_{ev}K_s$. Then, the total variation of momentum due to evaporation along the whole length of the wetted section is

$$\int_0^l V\,(x, t)\; 2\rho y K_{ev} K_s dx = 2\rho y K_{ev} K_s \left(\frac{2K_{ev}}{h\Pi} \frac{l^2}{2} + l \frac{dl}{dt} \right). \tag{5}$$

To determine the force of hydraulic resistance to liquid flow in cylindric capillaries we use the Poiseuille formula $Q = \left(\frac{\pi r^4}{8\eta}\right)\frac{\Delta P}{L}$ [3]. For N capillaries ($N = \frac{\Pi hy}{\pi r^2}$) we obtain $Q = \frac{r^2 \Delta P \Pi hy}{8L\eta}$. On the other hand, by definition $Q = \Pi hyV(x, t)$. Then, the force of hydraulic resistance for N capillaries of length $L = \Delta x$ is described as $f_{res} = \Delta P\Pi hy = V(x, t)8\eta\Delta x\frac{\Pi hy}{r^2}$ and the force of hydraulic resistance along the whole length l is expressed as

$$F_{res}\,(t) = \int_0^l V\,(x, t)\; 8\eta\, \frac{\Pi hy}{r^2}\, dx = 8\eta\, \frac{\Pi hy}{r^2} \left(\frac{2K_{ev}}{h\Pi} \frac{l^2}{2} + l \frac{dl}{dt} \right). \tag{6}$$

The motive force of the wetting process is the force of surface tension, which for one capillary of radius r has the form $f_{cap} = 2\sigma\pi r \cos\theta$. Then, the force moving liquid in N capillaries is presented as

$$F_{cap} = 2\sigma\Pi hy\, \frac{\cos\theta}{r}\,. \tag{7}$$

Substituting, with account for signs, (4)–(7) into (1), we obtain the equation

$$\frac{1}{2}\, l \frac{d^2 l}{dt^2} + \frac{1}{2}\left(\frac{dl}{dt}\right)^2 + \left(\frac{K_{ev}}{h\Pi} + \frac{4\eta}{\rho r^2} + \frac{K_{ev}K_s}{h\Pi} \right) l \frac{dl}{dt} + \left(\frac{4\eta K_{ev}}{\rho r^2 h\Pi} + \frac{K_{ev}^2 K_s}{h^2 \Pi^2} \right) l^2 = \sigma \frac{\cos\theta}{\rho r}\,, \tag{8}$$

which, in the absence of evaporation ($K_{ev} = 0$), gives the known formula presented in [4].

The components $0.5l\frac{d^2l}{dt^2}$ and $0.5\left(\frac{dl}{dt}\right)^2$ of formula (8) exert a considerable effect only on the initial sections of capillary soaking (at small l); in this case, the dynamic wetting angle, i.e., $\theta = \theta\left(\frac{dl}{dt}, l\right)$, must be taken into account. A detailed technique of calculation and results of experimental study of the dynamic wetting angle are given in [4]. For thin porous bodies the contribution of the terms $0.5l\frac{d^2l}{dt^2}$ and $0.5\left(\frac{dl}{dt}\right)^2$ is weakened due to the smallness of h. In the device developed, L_0 is selected such that dynamic variations of the wetting angle could be neglected. Then (8) is presented as

$$\left(\frac{K_{ev}}{h\Pi} + \frac{4\eta}{\rho r^2} + K_s \frac{K_{ev}}{h\Pi} \right) l \frac{dl}{dt} + \left(\frac{4\eta}{\rho r^2} \frac{K_{ev}}{h\Pi} + \frac{K_{ev}}{h\Pi} \frac{K_{ev}K_s}{h\Pi} \right) l^2 = \sigma \frac{\cos\theta}{\rho r}\,. \tag{9}$$

For real capillary-porous bodies, $K_s \le 1$, $\rho r^2 \ge h\Pi$, and the conditions $\frac{4\eta}{\rho r^2} >> 1 + K_s$ and $\frac{4\eta}{\rho r^2} >> \frac{K_{ev}K_s}{h\Pi}$ are satisfied; under these conditions the solution of (9)

$$l = \sqrt{\frac{\frac{\sigma\cos\theta}{\rho r}}{\frac{K_{ev}}{h\Pi}\left(\frac{4\eta}{\rho r^2}\frac{K_{ev}K_s}{h\Pi}\right)}}\sqrt{1 - \exp\left(-\frac{\frac{K_{ev}}{h\Pi}\left(\frac{4\eta}{\rho r^2} + \frac{K_{ev}K_s}{h\Pi}\right)}{\left(\frac{K_{ev}}{h\Pi} + \frac{4\eta}{\rho r^2} + K_s\frac{K_{ev}}{h\Pi}\right)}2t\right)}$$

is transformed to the form

$$l \approx \sqrt{\frac{r\sigma\cos\theta}{2\eta}}\sqrt{\frac{h\Pi}{2K_{ev}}}\sqrt{1 - \exp\left(-\frac{K_{ev}}{h\Pi}2t\right)}. \tag{10}$$

Substituting the distances that correspond to the position of the sensors in (10), we obtain

$$L_0 = \sqrt{\frac{r\sigma\cos\theta}{2\eta}}\sqrt{\frac{h\Pi}{2K_{ev}}}\sqrt{1 - \exp\left(-\frac{2K_{ev}}{\Pi h}t_0\right)}, \tag{11}$$

$$L_0 + L_1 = \sqrt{\frac{r\sigma\cos\theta}{2\eta}}\sqrt{\frac{h\Pi}{2K_{ev}}}\sqrt{1 - \exp\left(-\frac{2K_{ev}}{h\Pi}(t_0 + t_1)\right)}, \tag{12}$$

$$L_0 + L_1 + L_2 = \sqrt{\frac{r\sigma\cos\theta}{2\eta}}\sqrt{\frac{h\Pi}{2K_{ev}}}\sqrt{1 - \exp\left(-\frac{2K_{ev}}{h\Pi}(t_0 + t_1 + t_2)\right)}. \tag{13}$$

The model time t_0 in (11)–(13) can differ greatly from the real time due to the dynamic angle of wetting on the initial section and due to simplifications used in deriving (9). Therefore, in the device developed, only t_1 and t_2 are measured and the resultant formula for cos θ does not involve t_0. Obtaining the quantity $\exp\left(-\frac{2K_{ev}t_0}{h\Pi}\right)$ from Eqs. (11)–(13), we find

$$\exp\left(-\frac{2K_{ev}t_0}{h\Pi}\right) = \frac{(L_0 + L_1)^2 - L_0^2}{(L_0 + L_1)^2 - L_0^2\exp(-2K_{ev}t_1/(h\Pi))}, \tag{14}$$

$$\exp\left(-\frac{2K_{ev}t_0}{h\Pi}\right) = \frac{(L_0 + L_1 + L_2)^2 - L_0^2}{(L_0 + L_1 + L_2)^2 - L_0^2\exp(-2K_{ev}(t_1 + t_2)/(h\Pi))}, \tag{15}$$

$$\exp\left(-\frac{2K_{ev}t_0}{h\Pi}\right) = \frac{(L_0 + L_1 + L_2)^2 - (L_0 + L_1)^2}{(L_0 + L_1 + L_2)^2\exp(-2K_{ev}t_1/(h\Pi)) - (L_0 + L_1)^2\exp(-2K_{ev}(t_1 + t_2)/(h\Pi))}. \tag{16}$$

Any of Eqs. (14)–(16) allows elimination of t_0 from (10) by expressing $\exp\left(-\frac{2K_{ev}t_0}{h\Pi}\right)$ in terms of the parameters being determined; in this case, as follows from (14), it is enough to use only L_0, L_1, and t_1. The presence of L_2 and

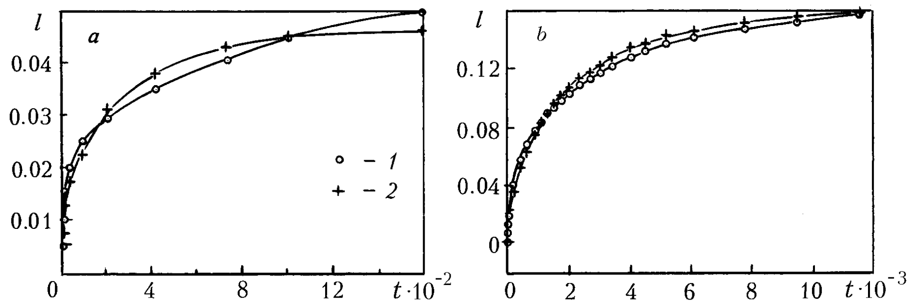

Fig. 2. Curves of propagation of the boundary of wetting: a) saturated soap solution; b) distilled water; 1) points of experimental curves; 2) points of approximating curves of propagation of the boundary of wetting.

t_2 allows one to take into account evaporation at constant K_{ev} during measurement. Equating the right-hand sides of (15) and (16), we obtain

$$\exp\left(-\frac{2K_{ev}}{h\Pi}(t_1+t_2)\right)=(L_1+L_2)\frac{2L_0+L_1+L_2}{L_1(2L_0+L_1)}\exp\left(-\frac{2K_{ev}}{h\Pi}t_1\right)-L_2\frac{2L_0+2L_1+L_2}{L_1(2L_0+L_1)}\,. \tag{17}$$

Numerical solution of Eq. (17) relative to the coefficients

$$\alpha=\frac{2K_{ev}}{h\Pi}\,, \tag{18}$$

which is obtained by the dichotomy method, helps to get rid of the necessity of separate determination of K_{ev}, h, and Π, since the result $l(t)$ depends on their ratio specified in the form of (18). Using the obtained coefficient α, we can write the equation that characterizes propagation of the boundary of wetting. With this in mind, we substitute (14) and (18) in (10) and obtain

$$l=\sqrt{\frac{r\sigma\cos\theta}{2\eta}}\sqrt{\frac{1}{\alpha}}\sqrt{1-\frac{(L_0+L_1)^2-L_0^2}{(L_0+L_1)^2-L_0^2\exp(-\alpha t_1)}\exp(-\alpha\Delta\tau)}\,. \tag{19}$$

At known values of σ, r, and η and measured values of times t_1 and t_2 of propagation of the boundary of wetting at L_1 and L_2, formula (19) allows determination of $\cos\theta$.

Formula (19) well approximated the experimental dependences of the boundary of wetting on time for different solutions. Approximating and experimental curves for different solutions are given in Fig. 2. The device makes it possible to determine the cosine of the wetting angle. The results obtained are in complete agreement with values of the wetting angle obtained by the Jurin formula $H = 2\sigma/(\rho g r)$ on the basis of the data of control tests on vertical ascent of liquid. When $\sigma = 0.07326$ N/m, $\rho = 1000$ kg/m^3, and $\eta = 0.001037$ Pa·sec on the filter paper with a mean pore radius $r = 4.25\cdot10^{-6}$ m and a height of ascent $H = 0.131$ m, the Jurin formula gives $\theta = 87.8638^{\circ}$, whereas according to (19) we have $\theta = 87.7612^{\circ}$.

NOTATION

F, sum of forces affecting liquid in the porous body, N; f_{res}, force of hydraulic resistance over the entire cross-sectional area on section Δx, N; F_{res}, force of hydraulic resistance over the entire cross-sectional area and length of the porous body, N; f_{cap}, surface-tension force in one capillary, N; F_{cap}, surface-tension force over the entire cross-sectional area of the porous body, N; H, height of liquid ascent in verification of results by the Jurin formula, m; h, height of the porous body, m; K_s, coefficient of proportionality between the mean velocity of liquid at the place of evaporation and the mean velocity of evaporating volume toward the $0x$ axis; K_{ev}, volume of liquid evaporating from the surface unit of the porous body in the time unit, m/sec; l, distance to which the boundary of wetting of the capillary body by liquid moves, m; L, length of a capillary, m; L_0, distance between the place of wetting and the sensor at the beginning of the first section, m; L_1 and L_2, first and second measurement sections, m; m, mass of liquid in the body volume, kg; $m_{\Delta x}$, mass of liquid on section Δx, kg; m_{en}, mass of liquid entering the body section, kg; m_{ev}, evaporated mass, kg; m_{out}, mass of liquid running out of the body section, kg; N, number of equivalent capillaries in the porous body with a mean radius r; $P(t)$, momentum of liquid in the body volume; Q, bulk flow rate, m^3/sec; r, capillary radius, m; t, current time, sec; t_0, t_1, and t_2, time of propagation of the boundary of wetting on the first, second, and third sections, sec; $V(x, t)$, velocity of liquid flow at a distance x from the contact point in time t, m/sec; x, current coordinate; y, width of the capillary-porous body, m; η, viscosity of liquid, Pa·sec; ρ, liquid density, kg/m^3; σ, surface tension of liquid, N/m; θ, wetting angle, deg; $\Delta\tau$, time reckoned from the moment when the boundary of wetting reaches the beginning of section L_1, sec; ΔP, pressure difference at the ends of the section, Pa; Π, bulk porosity, ratio of the volume of pores to the volume of the body. Subscripts: ev, evaporation; s, velocity; en, entering; out, outgoing; res, resistance; cap, capillary; 0, disregarded (discarded) time interval; 1 and 2, time intervals taken into account.

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